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FINAL REPORT

to the

Office of Naval Research

Naval Analysis Program

for

Contract N00014-77-C-0425

RANDOM PROCESS GENERATION

1 June 1977 -- 30 September 1979

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Bruce W. Schmeiser

Southern Methodist University

Dallas, TX 75275



Current Address:

Professor Bruce W. Schmeiser
School of Industrial Engineering

Purdue University

West Lafayette, IN 47907

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# ABSTRACT

This final report of the research resulting from contract N00014-77-C-0425 with the Office of Naval Research discusses the progress made in Random Process Generation and discusses the principal investigator's view of the future of the research area.

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#### 1. INTRODUCTION

This is the final report for Contract NO0014-77-C-0425 with the Naval Analysis Program of the Office of Naval Research. The initial title of the research, "General process generation for digital computer simulation," was revised to "Random process generation" on 1 January 1979 to reflect a broadened view of the research, which began on 1 June 1977 and ended on 30 September 1979. The contract ended because the principal investigator, Professor Bruce Schmeiser, left Southern Methodist University for Purdue University in the fall of 1979. The research continues at Purdue University under Contract N00014-79-C-0832 which began 1 October, 1979.

Random process generation includes all aspects of modeling and generating (pseudo)random variates for driving probabilistic simulations on digital computers. Topics include the selection of appropriate statistical distributions, parameter determination, diagnostic checking, and generation of random variables. The research performed under the auspices of this contract emphasized the use of general four-parameter families of distributions during the first year. The second year concentrated on generation of random vectors from multivariate distributions. Several other topics were considered as well, as discussed in Section 2.

The accomplishments of this research are described in Section 2.

Section 3 is a brief summary of the principal investigator's thoughts on the future of random process generation.

#### 2. ACCOMPLISHMENTS

We first list the technical reports, publications, and presentations related to this contract. A general discussion of the accomplishments follows.

- B. W. Schmeiser and M. Shalaby, "Acceptance/rejection methods for beta variate generation," <u>Journal of the American Statistical Association</u>, forthcoming.
- B. W. Schmeiser, "Methods for modelling and generating probabilistic components in digital computer simulation when the standard distributions are not adequate: A survey," <u>Proceedings of the Winter Simulation Conference</u>, National Bureau of Standards, Gaithersburg, MD, December 1977, 50-57.
  Reprinted in <u>Simuletter</u>, 10, 1 and 2 Fall 1978/Winter 1979, 38-43, 72.
- B. W. Schmeiser and M. Shalaby, "Rejection techniques in random variate generation using piecewise linear majorizing functions,"
  Proceedings of Computer Science and Statistics: Eleventh Annual
  Symposium on the Interface, May 1978, 230-233.
- B. W. Schmeiser and R. Lal, "Another versatile family of probability distributions," submitted for publication.
- B. W. Schmeiser, "Generation of variates from distribution tails,"

  Operations Research, forthcoming.
- B. W. Schmeiser, "Approximations to the inverse normal function for use on hand calculators," <u>Applied Statistics</u>, 28, 2 (1978), 175-176.

- B. W. Schmeiser and R. Lal, "Squeeze methods for gamma variate generation," <u>Journal of the American Statistical Association</u>, forthcoming.
- B. W. Schmeiser, "Order statistics in digital computer simulation:

  A survey," Proceedings of the Winter Simulation Conference,

  Miami, FL, December 1978, 136-140.
- B. W. Schmeiser and A. J. G. Babu, "Beta variates generation via exponential majorizing functions," Operations Research, forthcoming.
- B. W. Schmeiser and R. Lal, "Computer generation of bivariate gamma random vectors," Operations Research, subject to minor revision.
- B. W. Schmeiser and R. Lal, "Multivariate modeling in simulation: A survey," <u>Technical Conference Transactions</u>, American Society for Quality Control, Atlanta, GA, May 1980.

The following presentations related to this contract have been made.

All presentations were by the principal investigator, except where noted.

"Acceptance/rejection methods for beta variate generation," ORSA/TIMS

National Meeting, New York, NY, May 1978.

- "Methods for modeling and generating probabilistic components in digital computer simulation when the standard distributions are not adequate: A survey," 1978 Winter Simulation Conference.
- "Rejection techniques in random variate generation using piecewise linear majorizing function," Computer Science and Statistics:

  Eleventh Annual Symposium on the Interface, Raleigh, NC, May 1978.

- "Another versatile family of probability distributions," ORSA/TIMS
  National Meeting, New York, NY, May 1978.
- "Order statistics in digital computer simulation," 1978 Winter Simulation Conference, Miami, FL.
- "Computer generation of the four-parameter gamma distribution,"
  ORSA/TIMS National Meeting, Los Angeles, CA, November 1978.
- "Beta variate generation via exponential majorizing functions,"

  ORSA/TIMS National Meeting, New Orleans, LA, May 1979

  (presentation by A. J. G. Babu).
- "Computer generation of bivariate gamma random vectors," ORSA/TIMS
  National Meeting, New Orleans, LA, May 1979.
- "Continuous processes for input to digital computer simulation,"

  Office of Naval Research Conference on Simulation, Rougemont,

  NC, April 1979.
- -----, Department of Computer Science, McGill University,
  November 1979.
- ----, University of Montreal, November 1979.
- "Multivariate modeling in simulation: A survey," 33rd Annual

  Technical Conference of the American Society for Quality Control,

  Atlanta, GA, May 1980 (forthcoming).
- "Generation of continuous random variates: A state of the art survey,"

  ORSA/TIMS National Meeting, Milwaukee, WS, October 1979.

The original proposed research was to extend the family of distributions developed in Schmeiser and Deutsch, "A versatile family of probability distributions, suitable for simulation," AIIE Transactions, 9 (1977), 2, 176-182, to provide better modeling capability. The family, as published, was (and is) the only method for generating random variates having any specified first four moments through the use of a single function having only four parameters. The use of the single function simplified family selection, parameter determination and variate generation. However, it was hoped that generalization of the function would permit better fit to data. The poor fit was caused by a truncation on both tails and by the density function being either zero, one, or infinity at the mode (or antimode). The generalizations were developed, but the resulting family was so general (involving six parameters) that parameter fitting was made quite difficult. This work resulted in no technical report.

Being disappointed with the results of the first efforts, another tact was taken, which resulted in "A versatile family of distributions." This family has substantially better modeling properties, is easy to fit parameters to, is easy to generate random variates from, and can assume a wide range of shapes. It can not, however, match any given first four moments.

A second effort has been the development of algorithms for generating continuous univariate random variates from commonly used distributions.

Following the ideas outlined in Schmeiser and Shalaby (1978), methods for gamma and beta variate generation were developed in Schmeiser and Shalaby (JASA 1980), Schmeiser and Lel (JASA 1980), and Schmeiser and Babu (Operations Research 1980). These three papers also use ideas in Schmeiser (Operations

Research 1980) on distribution tails. The algorithms developed have marginal execution times which are faster than any other published algorithms by a factor of two. Of primary importance is that until 1978 there were no algorithms available which could generate gamma or beta variates exactly and with execution time which was insensitive to the parameter values of the distribution. In particular, as the gamma shape parameter goes to infinity and as the beta shape parameters go to infinity, the execution time required by algorithms available in 1978 went to infinity in the limit. At the same time the algorithms were being developed in this research, however, R. C. H. Cheng published gamma and beta variate algorithms which had robust execution times. The algorithms developed under this contract are about twice as fast as the best alternatives available today. Peter Lewis at the Naval Postgraduate School, who has implemented the algorithms in assembler language, has told the principal investigator that the new algorithms compare even more favorably in assembler language than in FORTRAN. Dr. James Gentle, in charge of statistical algorithms for the International Mathematical and Statistical Libraries, Inc. (Sixth Floor, GNB Building, 7500 Bellaire Boulevard, Houston, TX 77036), is implementing our algorithms G4PE (gamma generator) and B4PE (beta generator) into their version 8. This area of research has progressed very well.

The third area of research has been the development of algorithms for generating multivariate random vectors from continuous random variables.

Schmeiser and Lal (Operations Research 1980) devolope a family of algorithms for generating bivariate gamma vectors. These algorithms are the only methods available which are valid for any gamma marginal distributions and

for <u>any</u> desired correlations. Previous algorithms had limitations such as requiring the shape parameters of the marginal distributions to be integer, or to be equal, or for the correlation to be positive. Probably most importantly, the algorithm developed in this research allows regression curves to be modeled as well as the usual marginal distributions and correlation. Extensions to n dimensions are not straightforward, but it appears that we should soon have an n dimensional version of these algorithms.

#### 3. FUTURE DIRECTIONS

The methodologies available for the generation of random variates have been growing quickly during the last five or six years. Most of gain has been for univariate distributions, primarily in the use of acceptance/rejection algorithms for continuous distributions. Evaluating the past is straightforward: Here we discuss the less obvious future.

The principal investigator sees seven areas of variate generation will could receive considerable attention in the next few years.

### 1. Continuation of efforts to speed univariate variate generation.

It is not likely that faster algorithms for gamma and beta variates will be developed than those already developed in this reasearch. Also the normal distribution probably won't receive much attention. However, algorithms will be appearing for the lesser used distributions, such as inverse Gaussian, exponential power series, etc. Also the common discrete distributions will receive some attention, probably in the form of extending the use of the acceptance/rejection algorithm to the discrete case and extending the alias method of variate generation to allow for countably infinite ranges of the random variables.

### 2. Nonhomogeneous point processes.

The recent work of Peter Lewis and others in developing generation methods for nonhomogeneous Poisson point processes will probably be extended to nonPoisson processes. Similarly, Peter Lewis' development of time series models having nonnormal (usually gamma) marginal distributions will probably continue.

#### 3. Multivariate distributions.

The ideal here is to have an algorithm capable of generating n

dimensional random vectors having arbitrary marginal distributions (not identically distributed), with any theoretically possible correlation structure, and any theoretically possible regression curves. The bivariate gamma algorithms developed in this research are limited in that the marginal distributions must be gamma.

#### 4. Unification of the knowledge of variate generation.

More effort will be spent in trying to unify the field of variate generation. This is a field where relatively few concepts (say, less than fifteen) are used in various combinations to obtain new algorithms. The manner in these algorithms and concepts are similar and are different is understood only partially. For example, the alias/mixture/rejection method of Kronmal and Peterson (See, e.g. 1979 Proceedings of the Winter Simulation Conference.) is clearly closely related to various current algorithms, but it is not clear whether they are special cases, identical to, or a generalization of this method.

### 5. Random phenomena other than probability distributions.

Quite a bit of work will appear in the generation of random phenomena which are not in the form of probability distributions. For example, some recent work in probability has centered on the probabilistic properties of the area, perimeter, and number of points in random convex hulls (primarily in the computer science literature). It is of interest to estimate the mean, variance and other properties using Monte Carlo methods. The straight-forward algorithm is to generate n random points and to apply an algorithm to find the convex hull. However, since only about log (n) points are on the hull, this approach is inefficient for large numbers

of points, say n > 1000. It would be of interest to be able to generate the convex hull directly, or if that is not possible, to avoid generating most of the interior points.

# 6. Relationship between variate generation and variance reduction.

More attention will be paid to the interaction of the variate generation algorithm and the methods used to attempt variance reduction. For example, it is well-known that using the inverse transformation to generate a random variate X has the advantage of both synchronizing the variate generation (since exactly one U(0,1) variate is transformed to one value of X) and transforming some of the correlation structure of the uniform variates to the X's. However, since the inverse transformation is usually slow (particularly when numerical methods need to be used), there may be a large pay-off in developing methods of variate generation which still synchronize and transfer correlation structure, but which execute much faster. Also direct examination of the manner in which complicated variate generation algorithms can destroy the correlation structure necessary for variance reduction will be pursued.

### 7. Simulation on hand calculators.

The increasing use of hand calculators will make worthwhile methods for generating variates specifically designed to reduce the number of keys pressed rather than minimizing execution speed. An example is Schmeiser (Applied Statistics 1978).

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